

## THE DESIGN OF CABLE-STAYED BRIDGE WITH ELLIPTIC PYLON



**Seo, Jin-Hwan**  
Dept General Manager,  
Kunhwa Consulting and  
Engineering Co., Ltd.  
octopus8@snu.ac.kr



**Cho, Nam-Chul**  
Vice Chairman,  
Kunhwa Consulting and  
Engineering Co., Ltd.  
nccho@kunhwaeng.co.kr



**Seo, Dong-Kweon**  
Project Manager,  
Kunhwa Consulting and  
Engineering Co., Ltd.  
dkseo@kunhwaeng.co.kr



**Kim, Dong-Keun**  
Vice President,  
SK Engineering and  
Construction  
dkkim@skec.co.kr



**Rim, Ho-Sang**  
Vice President,  
Kunhwa Consulting and  
Engineering Co., Ltd.  
hsrim@kunhwaeng.co.kr



**Kim, Jae-Geum**  
Manager,  
SK Engineering and  
Construction  
jkkim-e@skec.co.kr

**Abstract:** The sector 4 of connecting road of the Incheon Bridge is passing by the artificial lake park connecting the Incheon Bridge to the 2<sup>nd</sup> Seoul-Incheon Expressway. The sector 4 is located at the centre of the lake park that is surrounded by Song-do International City and Song-do Amusement Park. So the aesthetic design is highly emphasized on because it can be viewed very near from those areas and closely exposed to the public. So SK E&C and Kunhwa Consortium tried its best to create new and unique structure during design competition and finally came to an elliptic pylon cable-stayed bridge. This paper will introduce the design concept of the main bridge briefly and study more about the structural behaviour of cable-stayed bridge with elliptic pylons.

**Keywords:** cable-stayed bridge, elliptic pylon, initial geometry, cable tuning, erection engineering, cable sag, nonlinearity of cable

### 1. Introduction

The sector 4 is located between The Incheon Bridge and Sector 5 connecting the Incheon Bridge to the 2<sup>nd</sup> Seoul-Incheon Expressway. And the main bridge of sector 4 is crossing over artificial lake park created by reclaimed land. The reclaimed land is now transforming to the International City



Figure 1: Geographical location

In terms of geographical location, the sector 4 holds key position as it is surrounded by the Song-do International Office and Residence Area and Song-do Amusement park and hinterland city of Nam-hang (South Port). So from the initial stage of design competition, it was focused on to make an aesthetically-pleasant and structurally-safe bridge.

The T.Y Lin International, advisory consulting firm, proposed 2 conceptual designs that meet the importance of key scenery area.

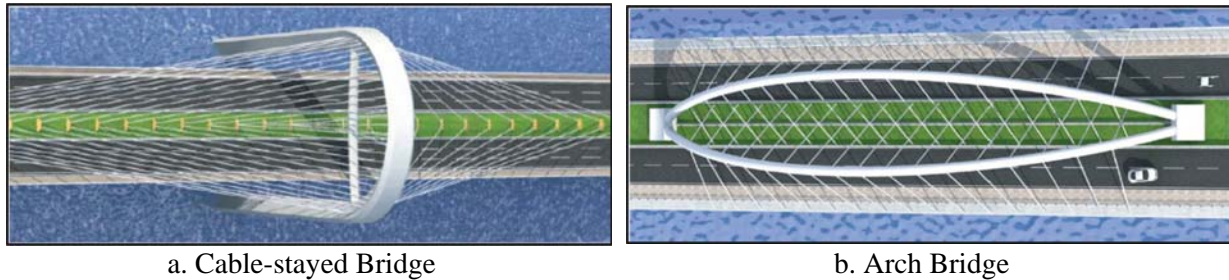


Figure 2: Conceptual Designs

Apparently, the cable-stayed bridge is a typical cable-stayed bridge with 1-pylon. But the stiffening girder is proposed as floating type to reduce the variation of pylon base moment and the cable arrangement is developed to reduce the in-plane moment of the top pylon arc. It forms very unique feature by the combinations of arc pylon top and cable arrangement. The second one, the arch bridge, is 'butterfly arch'. Two cables come from one point of each arch rib and those cables anchored to mid strip cross each other, which improves the buckling stability of the arch rib and lateral stiffness of girder.

Those designs were innovative and attractive ones. But wider width of main bridge and tapered width of approaching bridge are required which causes the increase of construction cost because the cables have to be anchored to the mid strip in both proposals. So our consortium proposes the new conceptual design of main bridge that can solve those problems and has unique figure as well. The design motif is 'Lotus flower on the calm lake'. The structural type is proposed as a cable-stayed bridge and to impart outstanding figure, the pylon is designed as oval shape.



Figure 3: Graphic image of the main bridge

## 2. Design of Cable-Stayed

### 2.1 Structural Plan

#### (1) Pylons

The sector 4 road is crossing over artificial lake and there is no ship passage, road or other things that have to be passing over. Because it doesn't require long span bridge, the structure type is determined to be small scale cable stayed bridge that is symmetric and 2-span continuous bridge with one pylon. Generally, 2-span continuous cable stayed bridge is vulnerable to fatigue and the deflection by live load (behaviour of live load) unless the pylon has enough stiffness. So as to impart enough fixity of vertical behaviour, the pylon usually becomes very massive.

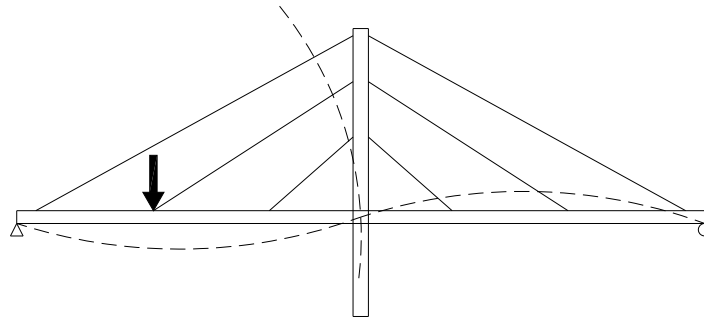


Figure 4: Structural Behaviour of one-pylon cable stayed bridge

In this project, the pylon is planned to be divided into 2 legs forming V-shape, expecting that this will give enough fixity of vertical deflection and slender impression considering that the main bridge is viewed near from the residence and office area. Because the V-shape pylon in combination with the cable system is similar to the warren truss system, the vertical deflection is very effectively limited, compared to that of the vertical pylon system.

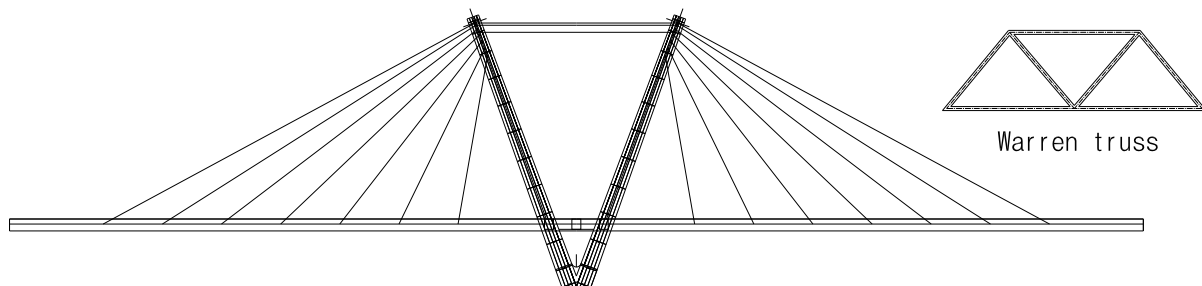


Figure 5: Similarity of warren truss system

Table 1: Example of the construction of one table

Item	Vertical Pylon system	V-Shape Pylon
Horizontal displacement ratio (Pylon Top)	1.62	1.00
Vertical displacement ratio (Girder)	Upward : 1.68 Downward: 1.35	Upward : 1.00 Downward: 1.00
Base Moment(Live Load)	1.23	1.00

From the front view, the pylon has an elliptic shape. In terms of structural efficiency, this shape will never be good choice because the large amount of bi-axial moment on the pylon section will be induced due to its oval shape. And furthermore the in-plane moment cannot be controlled by adjusting the tension force of cables. But considering that this main bridge is located at the centre of key scenery area, the technical solution will be searched to solve those structural problems that comes from oval shape. Those problems and solutions will be mentioned in more detail later in following paragraph.

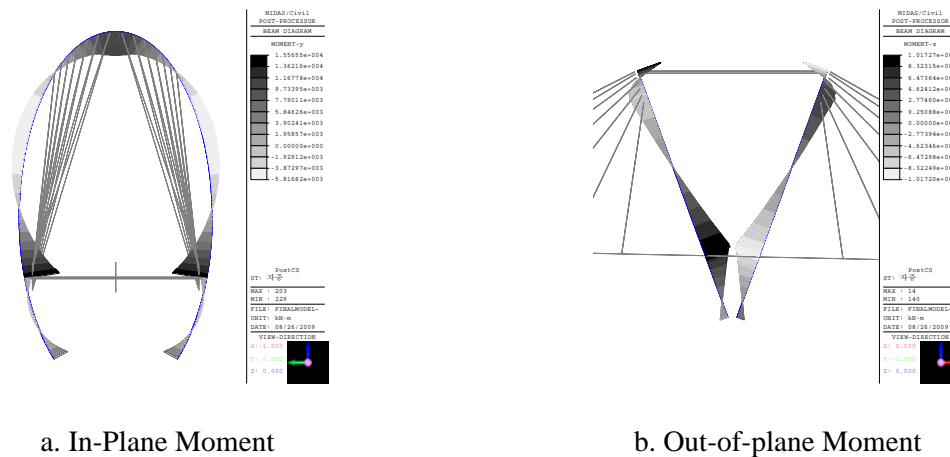


Figure 6: Biaxial Moment Directions

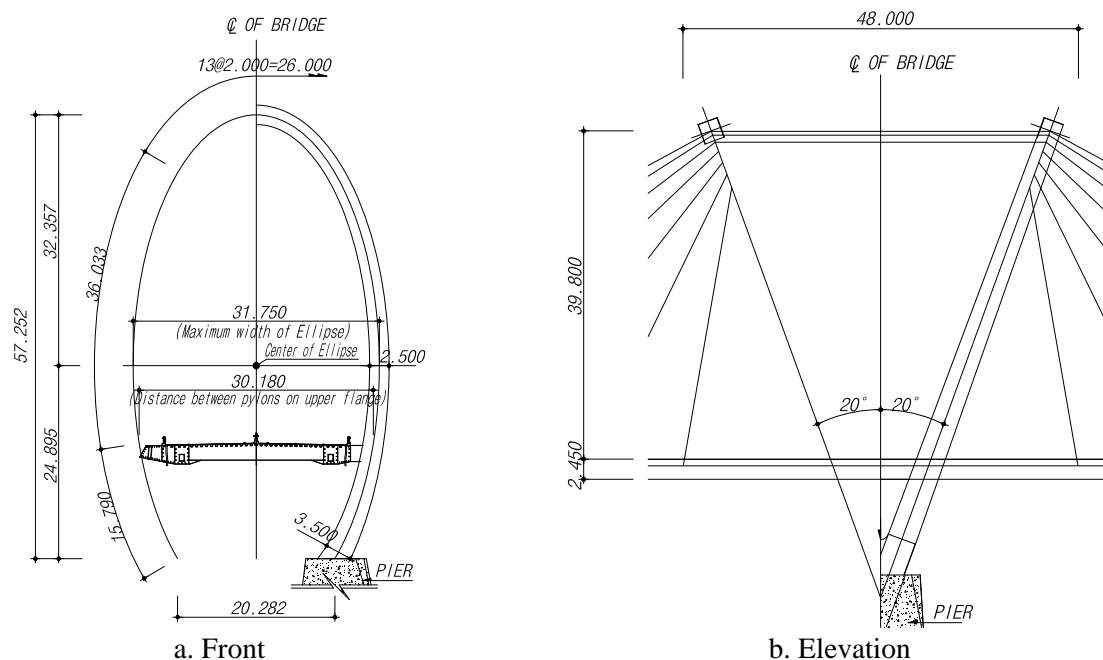


Figure 7: General Drawing (Pylon)

## (2) Stiffening Girder

In typical cable-stayed bridge, under dead load stiffening girder mainly takes axial force (the horizontal component of inclined cables) and the moment is very small. It is because the bending moment of girder is very similar to that of continuous girder having the cable anchorages as supports. But it is hardly possible to get the optimal cable forces with the conventional method because the elliptic pylon is too flexible to support the stiffening girder rigidly. So the design concept of stiffening girder is to make the girder take some part of dead load by bending stiffness and live load as well.



### ■ Deck System

Two types of stiffening girder system were studied at the preliminary design stage. R.C. deck system is preferred because of high cost of steel and gives the minimization of steelworks. (To minimize the amount of steelwork the R.C. deck system is compared to the steel deck system. In case of a girder bridge, R.C. composite deck system is more economical than the steel deck system.) Although the self-weight of superstructure is almost doubled by applying R.C deck composite system, the increase in steelwork is not doubled because the composite system will resist very effectively against post-composite loads. But from the analysis, the girder bending moment, normal stress at pylon, and cable tension force of composite system are 2.5~3 times larger than those of steel deck system under dead load. So the cable stayed bridge with flexible pylon can be designed more economically by reducing the self weight of stiffening girder.

Table 2: Comparison of Deck System

Items	Steel Deck	Concrete Deck (Composite)
Girder Bending Moment	1.00	1.32~2.75
Normal Stress at Pylon	1.00	2.38~2.45
Cable forces	1.00	2.15~3.02

### ■ Member Arrangement

Though the stiffness of pylon is relatively weak, the cable anchorage points on the girder system can be assumed to be elastically restrained. So basically all types of load that are imposed on the deck will be transferred to the edge girder through cross beams. From the load-carrying mechanism of the deck system, the mid stringer is omitted and the transverse ribs are closely spaced at every 2 metres and the cross beams are spaced at every 6 metres.

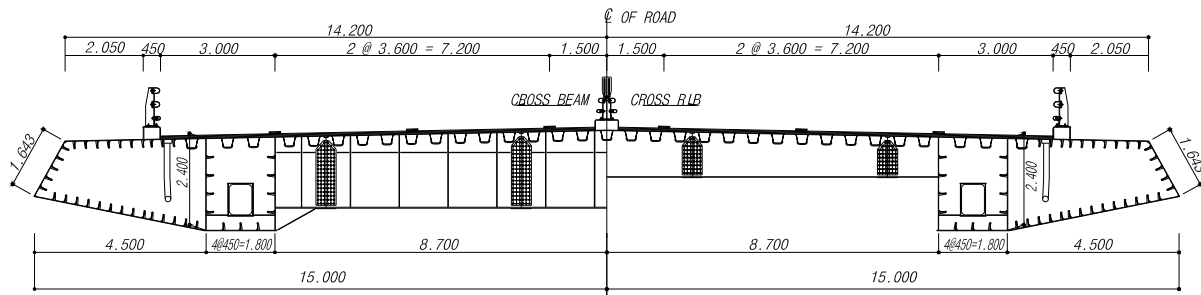


Figure 8: Stiffening Girder

## (3) Cables

### ■ Types of cable

There are two types of cable system, Prefabricated Wire Strand (P.W.S) and Multi Strand (M.S). For M.S type cable, the jacking device is small and strands reel can be easily handled because the cable is fabricated at a site (on-site condition) by iso-tensioning method. So if there is a space limitation for jacking( on the jacking space), it can be very advantageous. For P.W.S type, it is pre-fabricated from a factory(in the factory). Jacking device is huge because the target tension force has to be reached at once. Tension jacking force shall be less than 150 tons. Since live end of cable will be located beneath of lower flange of girder where there are plenty of rooms for stressing operation with false work platform, PWS

system shall be suitable for this bridge. (But this bridge is small scale cable stayed bridge and the tension force level is not higher than 150 ton. And because the live end of cable is beneath the lower flange of girder and the jacking work will be done on the false-work, there is no limitation on the working space. Therefore under these conditions the P.W.S type is more applicable, so the P.W.S system is adopted.)

### ■ Cable Arrangement

The cable arrangement cannot be determined with one-way procedure, which means that the cable arrangement and the initial state under dead load are mutually influencing. Therefore cable arrangement can be selected with the iterative procedure. Although some specific parameters related to the cable arrangement is determined through preliminary analysis results, those results can be different depending on the initial cable force under dead load. In this project, the cable spacing, the offset distance of first cable from girder end, and the numbers of horizontal cables are studied.

The cable spacing is less influencing on the structural system. The spacing is varied from 10m to 15m. As the cable is closely spaced, the positive bending moment is decreasing while negative bending moment is not influenced by cable spacing.( But the negative moment is not that different depending cable spacing.) Therefore (So) 12 metre spacing is adopted.

The offset distance of first cable from the girder end is determined as not to cause negative reactions (uplifting force). Generally, the cable (that is) anchored to the end support is called 'back-stay cable' to control the horizontal displacement of pylon as the independent variable. The cable force of this cable is directly transferred to the end support so the cable force is influencing on the top of pylon only and this requires tie-down devices to prevent uplifting force. For a small scale cable stayed bridge, the tie-down system is less preferable. So by varying the end offset distance from 10m to 19m, proper offset distance is searched to prevent uplifting force on bearings and 19m is turned out to be appropriate.

Table 3: Uplifting Reaction Check

Items	Dead Load (with cable tuning)	Live Load (with impact)
Vertical Reaction	173 ton	-40 ton
$D + 2 L (1+i)$	93 ton (Positive Reaction)	

The number of top tie cable is also studied. The first case is that the number of horizontal tie cables is same as the number of inclined cables. In this case the resultant forces is very effectively transferred to the pylon. The second case is to have 4 top tie cables which give the simple image from the side view. For the first case, as the out-of-plane movement is elastically restrained, the normal stress of pylon is reduced by(to) 40% compared to the second case. But because the tensioning work has to be done inside of pylon box which provides tiny space to workers(very small and narrow), the work environment(ability) is not pleasant. (expected to be quite bad). Therefore (So) the number of horizontal tie cable is determined to be 4.

### 2.3 Initial Geometry (Cable tuning)

As mentioned above the elliptic pylon is relatively flexible compared to straight pylon. So the conventional methods to determine the initial cable force **cannot be applied to** (are not true to) this system.

So the modified method has to be adopted. Basically to determine the optimal cable force, the 'Unknown factor module' of MIDAS/Civil is used. This module is to determine the cable forces for given boundary condition based on optimization theory. In terms of potential energy optimization, the 'Square' type object function is the most proper choice. But using the optimization theory, the stability of object function is very important. For the flexible structure like this, the correlation ratios between cable forces are very high which causes divergences for optimizing solution.

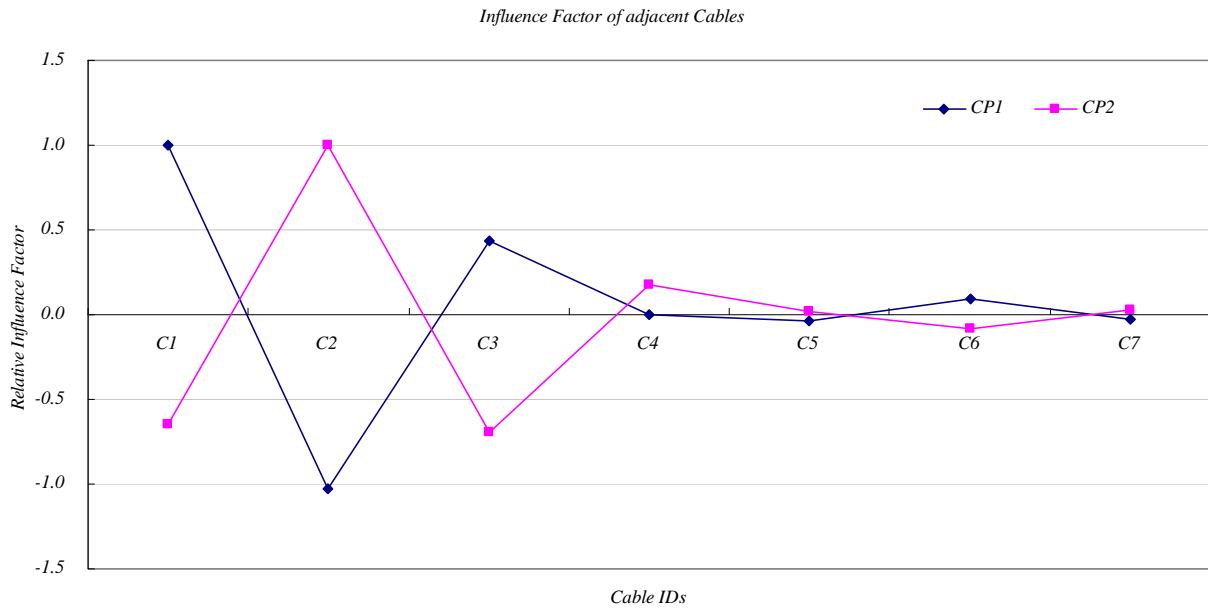


Figure 9: Influence factor of adjacent cables (Cable1&2)

Furthermore, there is no cable to reduce the in-plane moment of pylon (Figure10). Under this condition, the optimization solution will diverge because the strain energy of pylon in-plane moment only tends to increase unless the cable force is compression.

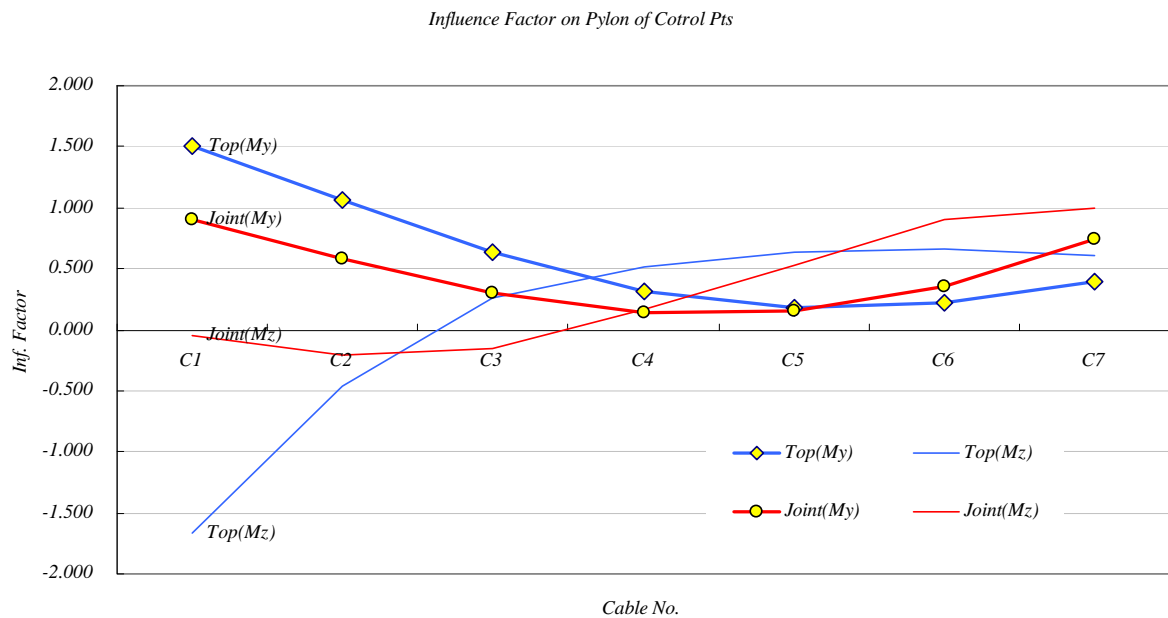


Figure 10: Influence factor of pylon biaxial moment

So as to decrease the influence of adjacent cable, the bending stiffness of girder is modified to 1/10 of prototype structure. As shown in Figure 11, the same result can be get irrespective of the object function type if the bending stiffness of girder is reduced to 1/10 of that of original structure. This will increase the solution stability and minimise the strain energy of pylons by intentionally devaluating the bending strain energy accumulated in the stiffening girder.

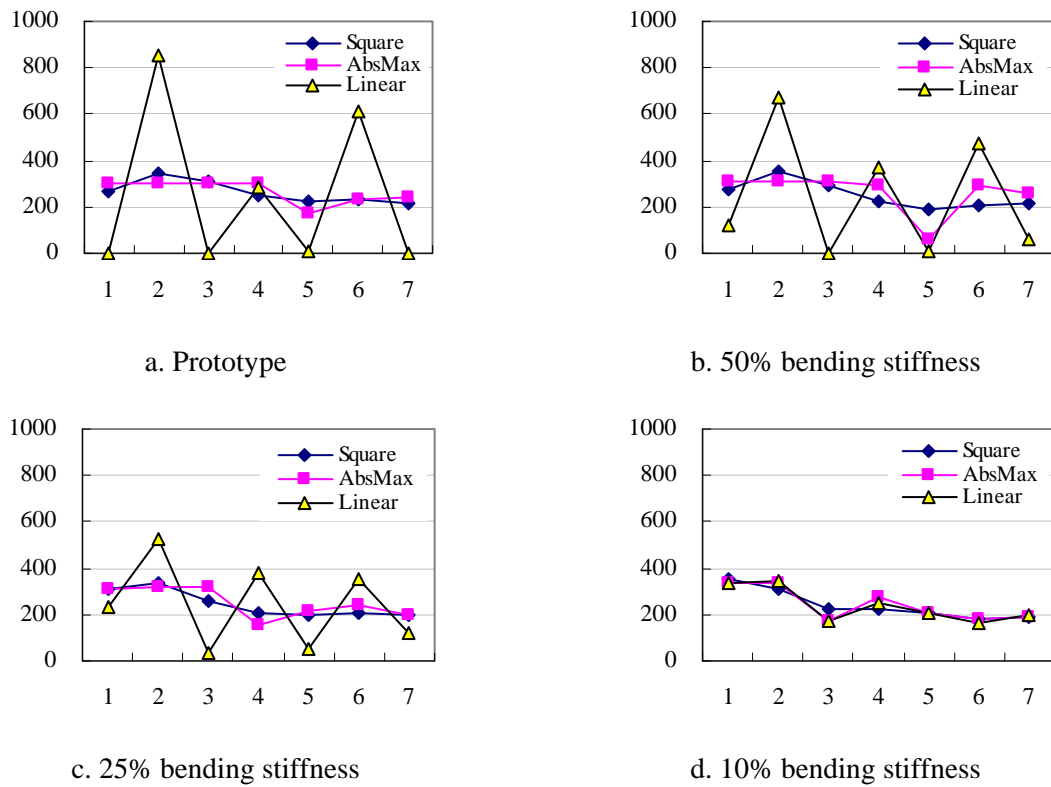


Figure 11: Solution Convergency (Stability of object function)

As seen in Figure 12, the strain energy induced by bi-axial bending in the pylon is decreased as the bending stiffness of girder is decreasing. This shows that the optimal cable force obtained under modified stiffness condition is the solution that minimises the total strain energy of structural system.



Figure 12: Strain Energy



## 2.4 Erection engineering

In cable stayed bridge design, the tension force that has to be introduced at the stage when the cable is installed should be suggested to fit the target force at the completion stage. For steel bridges which has no time-dependent characteristic, these unknown cable forces can be easily determined by backward analysis. If the forward analysis is carried out with those cable forces that are determined from backward analysis, both result will be almost same unless nonlinearity is that large. But if the bridge behaves nonlinearly due to the large cable sag, big compression forces, or large displacement etc, the results of forward analysis will be quite different from that of the completion stage.

The cable force history during construction stage from the backward analysis result is shown in Table 5. The result is for 1/4 of whole cables using symmetry.

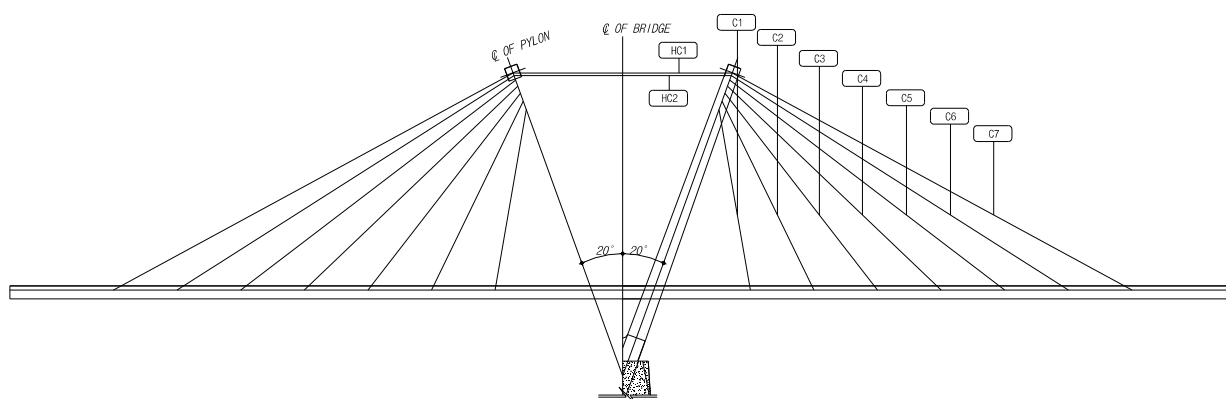


Figure 13: Cable Arrangement and number

Table 5: Cable Force History (in tonf)

STAGE	C1	C2	C3	C4	C5	C6	C7	HC 1	HC 2
01									
02									
03									
04									
05									
06									
07									
08									
09									23.367
10								12.953	11.711
11								12.972	11.725
12	33.633							19.882	18.031
13	30.405	45.871						33.629	30.753
14	27.151	40.830	54.377					52.589	48.671
15	23.469	34.969	47.915	71.892				79.256	74.505
16	19.121	27.452	40.338	64.995	99.064			117.305	112.339
17	14.601	19.398	31.975	57.112	91.190	123.537		165.079	161.034
18	10.238	11.532	23.312	48.487	82.964	115.370	140.712	218.419	216.371
19	66.941	122.012	136.217	143.965	152.975	159.267	154.246	415.140	404.804

For inclined cables (C1~C7), the nonlinearity due to cable sag is not that large during the construction stage because these cables are not that heavy and the initial tension force level goes up in proportional to the horizontal projection length. But the horizontal cables are relatively heavier than the inclines cables and the projection length is very long because they are aligned with horizontal line. So the tension force level is checked at the stage when those cables are activated. As seen in the table, the horizontal tie cables are activated at stage 9, 10 in series and the cable forces are 12.953 ton and 23.367 ton each. The tangential stiffness is calculated in Table 6.

Table 6: Tangential elastic modulus of horizontal cables

Items	Area (cm <sup>2</sup> )	Projection Length(m)	Unit weight (kg/m)	Cable Force (tonf)	Sag(m)	Tangential Modulus (kgf/cm <sup>2</sup> )	Etan/Eo
HC1	76.584	39.478	64.2	12.953	0.996	51,644	2.58 %
HC2	76.584	39.822	64.2	23.367	0.545	265,296	13.26 %

As seen in Table 6, the tangential elastic modulus is only 2.58% and 13.26% of the original elastic modulus which means that the structure will experience large nonlinearity during the construction stage and cause the member forces in structure to be different from what is intended at the completion stage.

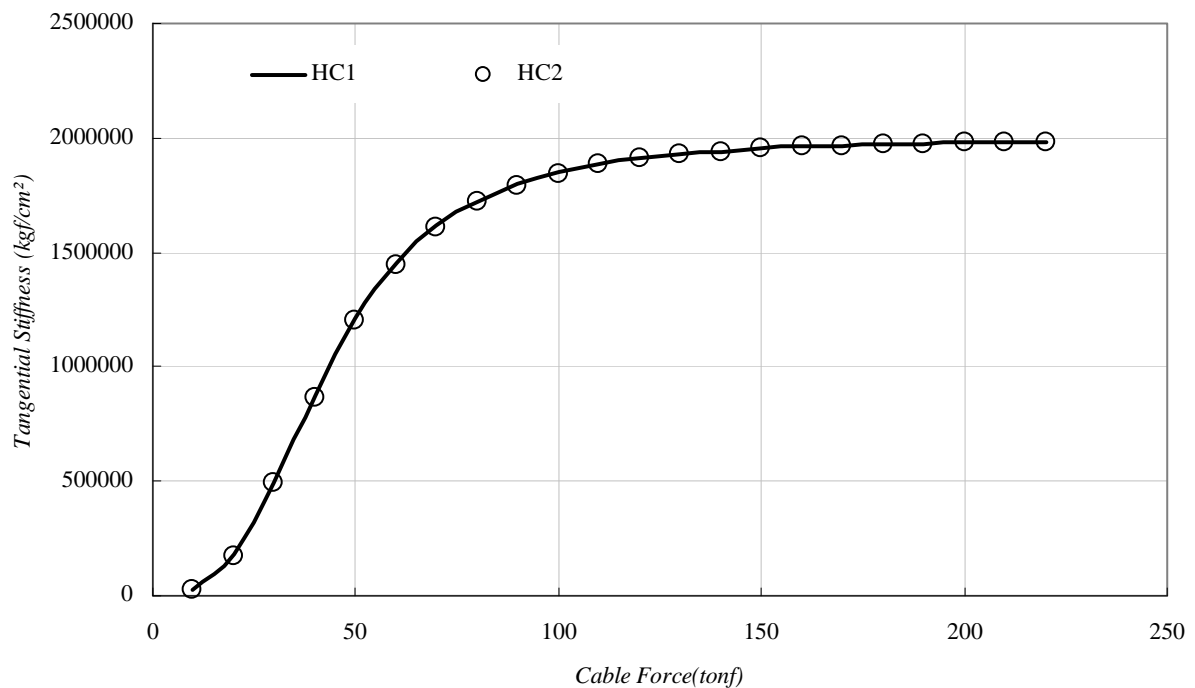


Figure 14: Tangential elastic modulus of horizontal cables

When this problem first was found, the use of temporary beam under horizontal cable was considered. Because the cable sag nonlinearity is caused by the deflection of cable, the nonlinearity can be successfully controlled if the self-weight of cable is properly supported. But this method needs 40-metre auxiliary beams which are very long and might cause instability of temporary member itself due to temperature load or wind load. Therefore the cable force jacking order and force level have to be changed to minimize the nonlinearity due to cable sag during the construction stage. Figure 14 shows the

variation of tangential elastic modulus of horizontal cables. From the graph, the force level should be kept at least 90 tonf during the construction stage to maintain the tangential elastic modulus not lower than 90% of the original elastic modulus. Based on the analysis result, the cable force level and jacking order **are** modified and the newly adjusted order is shown in Table 7.

Table 7: Revised cable force history (in tonf)

STAGE	C1	C2	C3	C4	C5	C6	C7	HC 1	HC 2
01									
02									
03									
04									
05									
06									
07									
08									
09									90.000
10									89.980
11	40.000								105.349
12	34.977	55.000							136.001
13	29.506	46.088	68.000						179.084
14	23.227	35.563	56.353	90.000					238.196
15	28.822	46.694	68.699	102.161				135.000	140.045
16	23.997	38.403	60.315	94.502	109.000			177.248	181.798
17	19.749	30.882	52.481	87.091	101.607	115.000		222.107	227.235
18	15.989	24.146	45.044	79.664	94.528	107.984	120.000	267.970	274.513
19	66.093	120.774	144.718	165.655	159.068	149.889	134.041	446.581	444.786

As seen in Table 7, the horizontal cable forces is not lower than 90 tonf which means that the tangential modulus of horizontal cable will be over 90% of original modulus during construction stage. So if following this jacking order and force, the final structure will have the same member force as it is targeted.

Though the nonlinear behaviour is restricted by maintaining the level of cable forces, there are still some differences between linear analysis result and nonlinear one. So it is necessary to verify the validity of linear analysis result by nonlinear analysis comparison. In nonlinear analysis, the geometry nonlinearity and nonlinearity by cable sag is taken into account by using 'Cable Element'. And the material nonlinearity will be excluded.

Table 8: Cable forces at the completion (in tonf)

STAGE	C1	C2	C3	C4	C5	C6	C7	HC 1	HC 2
Linear	66.093	120.774	144.718	165.655	159.068	149.889	134.041	446.581	444.786
Nonlinear	66.198	121.418	145.184	165.978	159.341	150.116	134.065	446.008	445.517
Error (%)	0.16	0.53	0.32	0.19	0.17	0.15	0.02	0.13	0.16

The differences of cable forces between linear analysis and nonlinear analysis range from 0.02% to 0.53% only. It can be ignored in design practice and the new construction stage is well assumed to ignore those nonlinearity effects. The member force of pylon is also compared at those sections where the member force is highly concentrated-top and middle.

Table 9: Pylon member force at the completion

Position	Items	Axial (tonf)	Shear-y (tonf)	Shear-z (tonf)	Torsion (tonf·m)	Moment-y (tonf·m)	Moment-z (tonf·m)
Middle	Linear	-784.18	1.92	-11.45	36.57	-1245.67	-1308.20
	Nonlinear	-785.00	1.80	-11.34	39.34	-1250.99	-1351.01
	Error (%)	0.10	6.25	0.96	7.57	0.43	3.27
Top	Linear	-62.10	0.76	2.07	151.71	2668.05	2261.63
	Nonlinear	-61.30	0.70	1.98	153.80	2679.15	2268.61
	Error (%)	1.29	7.89	4.35	1.38	0.42	0.31

The member force components that cause normal stress on section- axial, moment-y and moment-z- are not showing big differences between the linear and nonlinear result whereas in the shear force and the torsion moment, the error amounts to about 8%. But the shear and the torsion component is not decisive factor to determine section dimension and the absolute value is very low so the errors will have minor effects on the whole structure. But to consider the nonlinearity effects on these force component, the additional margin have to be considered.

### 3. Conclusion

In this paper, the behaviour characteristic of cable stayed bridge with elliptic pylons is briefly reviewed. Different from general cable stayed bridge, some special considerations are necessary for determining the optimal cable forces. And in most large-scale cable stayed bridge, the tension force level is high enough to guarantee the linear behaviour. But for small scale bridge like this project, the tension force level is critical so the nonlinear behaviour has to be investigated carefully.

### REFERENCES

- [1] Korea Bridge Design Code (2005).
- [2] Design guide of Steel Cable bridges (2006)
- [3] An introduction to optimization (3<sup>rd</sup> Edition), Chong, Edwin Kah Pin, Zak, Stainslaw H., John and Wiley Sons Inc.
- [4] MIDAS/Civil Program Manual